Introduction

The theory of sets was developed by German mathematician Georg Cantor (1845-1918).

A set is a well-defined collection of objects.

N : the set of all natural numbers
Z : the set of all integers
Q : the set of all rational numbers
R : the set of real numbers
Z+ : the set of positive integers
Q+ : the set of positive rational numbers, and
R+ : the set of positive real numbers.

There are two methods of representing a set :
(i) Roster or tabular form
(ii) Set-builder form.

(i) In roster form, all the elements of a set are listed, the elements are being separated by commas and are enclosed within braces { }.
For example, the set of all even positive integers less than 7 is described in roster form as {2, 4, 6}.

(ii) In set-builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set.
For example, in the set $\{a, e, i, o, u\}$, all the elements possess a common property, namely, each of them is a vowel in the English alphabet, and no other letter possess this property. Denoting this set by V, we write V = $\{x : x$ is a vowel in English alphabet$\}$

Example 1: Write the following set in roster form: C = $\{x$: x is a two-digit natural number such that the sum of its digits is 8$\}$
Solution: C = $\{x$: x is a two-digit natural number such that the sum of its digits is 8$\}$
The elements of this set are 17, 26, 35, 44, 53, 62, 71, and 80 only.
Therefore, this set can be written in roster form as C = {17, 26, 35, 44, 53, 62, 71, 80}

Example 2: Write the following set in the set-builder form: {1, 4, 9 … 100}
Solution: It can be seen that $1 = 1^2$, $4 = 2^2$, $9 = 3^2$ …$100 = 10^2$.
$\therefore \{1, 4, 9 \ldots 100\} = \{x: x = n^2, n \in \mathbf{N} \text{ and } 1 \le n \le 10\}$

The Empty Set

A set which does not contain any element is called the *empty set* or the *null set* or the *void set*.

For example, A = $\{ x : x$ is a student presently studying in both Classes X and XI $\}$

Finite and Infinite Sets

A set which is empty or consists of a definite number of elements is called *finite* otherwise, the set is called *infinite*.
For example :
(i) Let W be the set of the days of the week. Then W is finite.
(ii) Let S be the set of solutions of the equation $x^2 - 16 = 0$. Then S is finite.
(iii) Let G be the set of points on a line. Then G is infinite.

Example 3: State which of the following sets are finite or infinite:
(i) $\{x : x \in N \text{ and } (x - 1)(x - 2) = 0\}$
(ii) The set of positive integers greater than 100
Solution: (i) Given set = {1, 2}. Hence, it is finite.
(ii) The set of positive integers greater than 100 is an infinite set because positive integers greater than 100 are infinite in number.

Equal Sets

Two sets A and B are said to be *equal* if they have exactly the same elements and we write A = B. Otherwise, the sets are said to be *unequal* and we write $A \neq B$.
We consider the following examples :
(i) Let A = {1, 2, 3, 4} and B = {3, 1, 4, 2}. Then A = B.
(ii) Let A be the set of prime numbers less than 6 and P the set of prime factors of 30. Then A and P are equal, since 2, 3 and 5 are the only prime factors of 30 and also these are less than 6.

Example 4: Which of the following pairs of sets are equal? Justify your answer.
(i) X, the set of letters in "ALLOY" and B, the set of letters in "LOYAL".
(ii) A = {2, 3}; B = $\{x: x \text{ is solution of } x^2 + 5x + 6 = 0\}$
Solution: (i) We have, X = {A, L, L, O, Y}, B = {L, O, Y, A, L}. Then X and B are equal sets as repetition of elements in a set do not change a set. Thus, X = {A, L, O, Y} = B
(ii) A = {2, 3}; B = $\{x: x \text{ is a solution of } x^2 + 5x + 6 = 0\}$. The equation $x^2 + 5x + 6 = 0$ can be solved as:
$x(x + 3) + 2(x + 3) = 0$
$(x + 2)(x + 3) = 0$
$x = -2$ or $x = -3$
$\therefore A = \{2, 3\}; B = \{-2, -3\}$
$\therefore A \neq B$

Example 5: Make correct statements by filling in the symbols $\subset$ or $\not\subset$ in the blank spaces:
(i) $\{x: x$ is a student of Class XI of your school$\}$ … $\{x: x$ student of your school$\}$
(ii) $\{x: x$ is a triangle in a plane$\}$…$\{x: x$ is a rectangle in the plane$\}$
Solution: (i) $\{x: x$ is a student of class XI of your school$\} \subset \{x: x$ is student of your school$\}$
(ii) $\{x: x$ is a triangle in a plane$\} \not\subset \{x: x$ is a rectangle in the plane$\}$

Subsets

A set A is said to be a subset of a set B if every element of A is also an element of B.

In other words, A $\subset$ B if whenever $a \in$ A, then $a \in$ B. It is often convenient to use the symbol "$\Rightarrow$" which means *implies*. Using this symbol, we can write the definition of *subset* as follows:
A $\subset$ B if $a \in$ A $\Rightarrow$ $a \in$ B
We read the above statement as "*A is a subset of B if a is an element of A implies that a is also an element of B*". If A is not a subset of B, we write A $\not\subset$ B.

We may note that for A to be a subset of B, all that is needed is that every element of A is in B. It is possible that every element of B may or may not be in A. If it so happens that every element of B is also in A, then we shall also have B $\subset$ A.
In this case, A and B are the same sets so that we have A $\subset$ B and B $\subset$ A $\Leftrightarrow$ A = B, where "$\Leftrightarrow$" is a symbol for two way implications, and is usually read as *if and only if* (briefly written as "iff").

It follows from the above definition that every set *A is a subset of itself*, i.e., A $\subset$ A. Since the empty set $\emptyset$ has no elements, we agree to say that φ *is a subset of every set*.

Intervals as subsets of R

Let $a, b \in$ **R** and $a < b$. Then the set of real numbers $\{ y : a < y < b\}$ is called an *open interval* and is denoted by (a, b). All the points between a and b belong to the open interval (a, b) but a, b themselves do not belong to this interval.

The interval which contains the end points also is called *closed interval* and is denoted by $[a, b]$. Thus
$[a, b] = \{x : a \leq x \leq b\}$

We can also have intervals closed at one end and open at the other, i.e.,
$[a, b) = \{x : a \leq x < b\}$ is an *open interval* from a to b, including a but excluding b.
$(a, b] = \{ x : a < x \leq b \}$ is an *open interval* from a to b including b but excluding a.

Power Set

The collection of all subsets of a set A is called the *power set* of A.

It is denoted by P(A). In P(A), every element is a set.
Thus, if A = { 1, 2 }, then P(A) = { φ,{ 1 }, { 2 }, { 1,2 }}

In general, if A is a set with n(A) = m, then it can be shown that n [P(A)] = 2^m.

Universal Set

While studying the system of numbers, we are interested in the set of natural numbers and its subsets such as the set of all prime numbers, the set of all even numbers, and so forth. This basic set is called the "*Universal Set*".
The universal set is usually denoted by U, and all its subsets by the letters A, B, C, etc.

For example, for the set of all integers, the universal set can be the set of rational numbers or, for that matter, the set **R** of real numbers.

Venn Diagrams

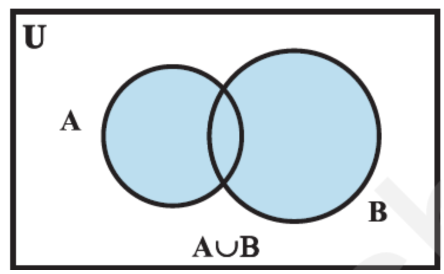
The relationships between sets can be represented by means of diagrams which are known as *Venn diagrams*. Venn diagrams are named after the English logician, John Venn (1834-1883).

These diagrams consist of rectangles and closed curves usually circles. The universal set is represented usually by a rectangle and its subsets by circles.

Operations on Sets

1. Union of sets

Let A and B be any two sets. The union of A and B is the set which consists of all the elements of A and all the elements of B, the common elements being taken only once. The symbol '∪' is used to denote the *union*. *Symbolically, we write* A ∪ B *and usually read as 'A union B'*.



Example 6: Let X = {Ram, Geeta, Akbar} be the set of students of Class XI, who are in school hockey team. Let Y = {Geeta, David, Ashok} be the set of students from Class XI who are in the school football team. Find X ∪ Y and interpret the set.
Solution We have, X ∪ Y = {Ram, Geeta, Akbar, David, Ashok}. This is the set of students from Class XI who are in the hockey team or the football team or both.

Example 7: Find the union of the following pairs of sets: A = {x: x is a natural number and multiple of 3}; B = {x: x is a natural number less than 6}.

Solution: A = {x: x is a natural number and multiple of 3} = {3, 6, 9 …}
As B = {x: x is a natural number less than 6} = {1, 2, 3, 4, 5, 6};
A ∪ B = {1, 2, 4, 5, 3, 6, 9, 12 …};
∴ A ∪ B = {x: x = 1, 2, 4, 5 or a multiple of 3}

Some Properties of the Operation of Union

$(i)\ A \cup B = B \cup A\ (Commutative\ law)$

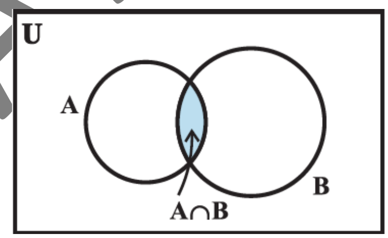
$(ii)\ (A \cup B) \cup C = A \cup (B \cup C)$
$(Associative\ law)$
$(iii)\ A \cup \emptyset = A\ (Law\ of\ identity\ element, \emptyset\ is\ the\ identity\ of\ \cup)$
$(iv)\ A \cup A = A\ (Idempotent\ law)$
$(v)\ U \cup A = U\ (Law\ of\ U)$

2. Intersection of sets

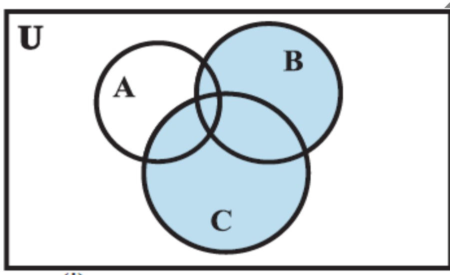
The intersection of sets A and B is the set of all elements which are common to both A and B. The symbol ∩ ' is used to denote the intersection. The intersection of two sets A and B is the set of all those elements which belong to both A and B. Symbolically, we write $A \cap B = \{x : x \in A \text{ and } x \in B\}$.

Example 8: Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and B = { 2, 3, 5, 7 }.
Find A ∩ B and hence show that A ∩ B = B.
Solution: We have A ∩ B = { 2, 3, 5, 7 } = B. We note that B ⊂ A and that A ∩ B = B.

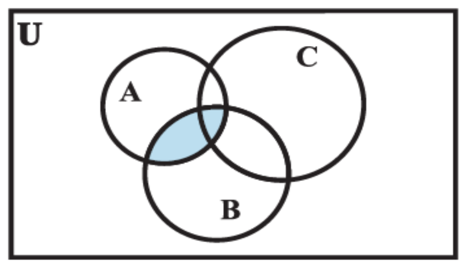
Some Properties of Operation of Intersection



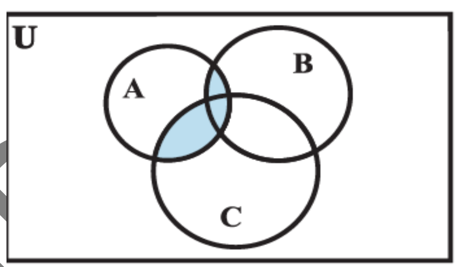
$(i)\ A \cap B = B \cap A\ (Commutative\ law)$.
$(ii)\ (A \cap B) \cap C = A \cap (B \cap C)\ (Associative\ law)$.
$(iii)\ \emptyset \cap A = \emptyset, U \cap A = A\ (Law\ of\ \emptyset\ and\ U)$.
$(iv)\ A \cap A = A\ (Idempotent\ law)$
$(v)\ A \cap (B \cup C) = (A \cap B) \cup (A \cap C)\ (Distributive\ law)\ i.e., \cap\ distributes\ over\ \cup$



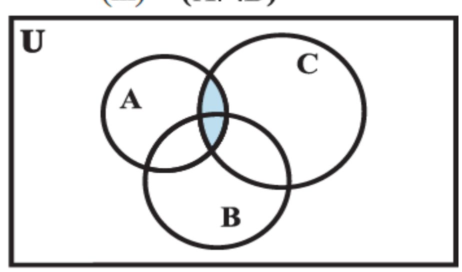
(i) (B∪C)



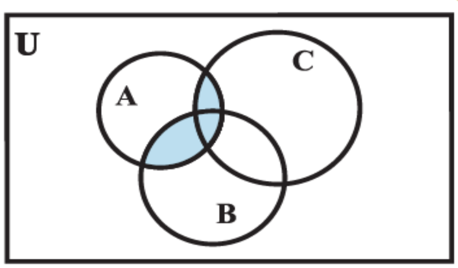
(iii) (A∩B)



(ii) A∩(B∪C)



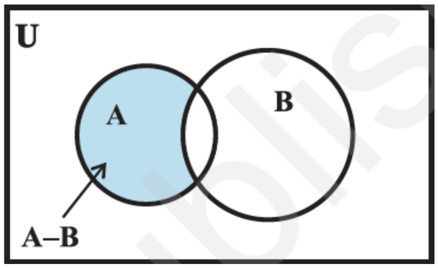
(iv) (A∩C)

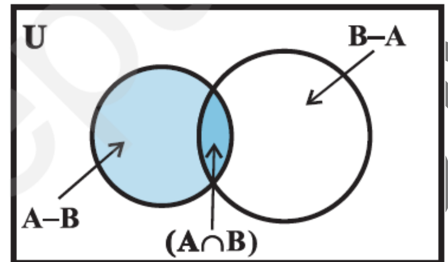


(v) (A∩B) ∪ (A∩C)

Difference of sets

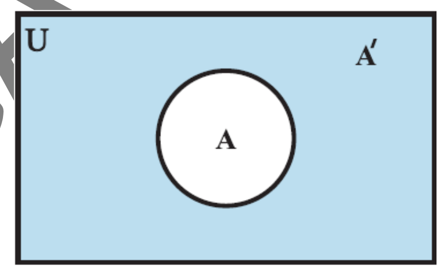
The difference of the sets A and B in this order is the set of elements which belong to A but not to B. Symbolically, we write A – B and read as “ A minus B”.





Complement of a Set

Let U be the universal set and A a subset of U. Then the complement of A is the set of all elements of U which are not the elements of A.



Symbolically, we write A' to denote the complement of A with respect to U. Thus,
$A' = \{x : x \in U \ and \ x \notin A\}. Obviously \ A' = U - A$

De Morgan’s laws

The complement of the union of two sets is the intersection of their complements and the complement of the intersection of two sets is the union of their complements. These are called De Morgan’s laws.

Some Properties of Complement Sets

1. Complement laws: (i) $A \cup A' = U$ (ii) $A \cap A' = \emptyset$
2. De Morgan’s law: (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$
3. Law of double complementation : $(A')' = A$
4. Laws of empty set and universal set $\emptyset' = U \ and \ U' = \emptyset$.

Union and Intersection of Two Sets

1. If A and B be finite sets. If $A \cap B = \emptyset$, then n (A ∪ B) = n (A) + n (B)

2. If A and B are finite sets, then n (A ∪ B) = n (A) + n (B) – n ($A \cap B$)
3. If A, B and C are finite sets, then
n (A ∪ B ∪ C) = n (A) + n (B) + n (C) – n (A ∩ B) – n (B ∩ C) – n (A ∩ C) + n (A ∩ B ∩ C)

Example 9: Is the following pair of sets is disjoint: {1, 2, 3, 4} and {*x*: *x* is a natural number and $4 \le x \le 6$}

Solution: {1, 2, 3, 4}, {*x*: *x* is a natural number and $4 \le x \le 6$} = {4, 5, 6}

Now, {1, 2, 3, 4} ∩ {4, 5, 6} = {4}

Therefore, this pair of sets is not disjoint.

Example 10: If X = {*a*, *b*, *c*, *d*} and Y = {*f*, *b*, *d*, *g*}, find (i) X – Y (ii) Y – X (iii) X ∩ Y

Solution: (i) X – Y = {*a*, *c*} (ii) Y – X = {*f*, *g*} (iii) X ∩ Y = {*b*, *d*}

Example 11: If U = {1, 2, 3, 4, 5,6,7,8, 9}, A = {2, 4, 6, 8} and B = {2, 3, 5, 7}. Verify that

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

Solution: (i)

$$(A \cup B)' = \{2, 3, 4, 5, 6, 7, 8\}' = \{1, 9\}$$
$$A' \cap B' = \{1, 3, 5, 7, 9\} \cap (1, 4, 6, 8, 9) = \{1, 9\}$$
$$\therefore (A \cup B)' = A' \cap B'$$

(ii)

$$(A \cap B)' = \{2\}' = \{1, 3, 4, 5, 6, 7, 8, 9\}$$
$$A' \cup B' = \{1, 3, 5, 7, 9\} \cup \{1, 4, 6, 8, 9\} = \{1, 3, 4, 5, 6, 7, 8, 9\}$$
$$\therefore (A \cap B)' = A' \cup B'$$

Example 12: In a group of 70 people, 37 like coffee, 52 like tea, and each person likes at least one of the two drinks. How many people like both coffee and tea?

Solution: Let C denote the set of people who like coffee, and T denote the set of people who like tea

$n(C \cup T) = 70$, $n(C) = 37$, $n(T) = 52$

We know that:

$n(C \cup T) = n(C) + n(T) - n(C \cap T)$

$\therefore 70 = 37 + 52 - n(C \cap T)$

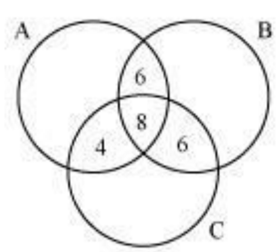
$\Rightarrow 70 = 89 - n(C \cap T)$

$\Rightarrow n(C \cap T) = 89 - 70 = 19$

Thus, 19 people like both coffee and tea.

Example 13: In a survey it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B, 12 people liked products C and A, 14 people liked products B and C and 8 liked all the three products. Find how many liked product C only.

Solution: Let A, B, and C be the set of people who like product A, product B, and product C respectively. Accordingly, $n(A) = 21$, $n(B) = 26$, $n(C) = 29$, $n(A \cap B) = 14$, $n(C \cap A) = 12$, $n(B \cap C) = 14$, $n(A \cap B \cap C) = 8$. The Venn diagram for the given problem can be drawn as



It can be seen that number of people who like product C only is

{29 – (4 + 8 + 6)} = 11